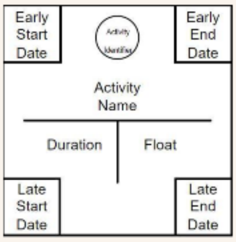
net value function

$$\text{NVF} = \text{benefit} - \text{cost}$$



conversion factors

marginal value change.

> extra net value obtained for one more item

$$\Delta NV = NV(x+1) - NV(x)$$

optimisation

model-based

- conclusions from the model of the system

Components:

- decision variables
- constraints
- objectives
- functions: mathematical function that determines the objective as a function of decision variable

$$\begin{aligned} \min_x \phi = f(x) & \quad \leftarrow \text{Objective function} \\ \text{s.t} & \quad \leftarrow \text{Constraints} \\ h(x) = 0 & \quad \leftarrow \text{Equality constraints} \\ g(x) \leq 0 & \quad \leftarrow \text{Inequality constraints} \end{aligned}$$

constraints

- physical limitations: cannot purchase negative raw materials
- model assumptions: assumptions about the system

domain of a definition
a decision upper and lower bounds ($x^{\mathcal{U}}$ and $x^{\mathcal{L}}$)

Properties

- **Active/binding:** $\exists\, x^* \mid g(x^*) = 0$
- **Inactive:** $\exists\, x^* \mid g(x^*) < 0$

graphing models

feasible set of an optimization model
The collection of decision variables that satisfy all constraints

$$\mathcal{S} \triangleq \{x : g(x) \leq 0, h(x) = 0, x^L \leq x \leq x^U\}$$

outcomes

optimal value
the optimal value ϕ^* is the value of the objective at the optimum(s)

$$\phi^* \triangleq \phi(x^*)$$

> Constraints satisfy, but it is not binding

Linear optimization problems

$$\begin{aligned} \min_{x_1, x_2} \phi = 50x_1 + 37.5x_2 \\ \text{s.t} \\ 0.3x_1 + 0.4x_2 \geq 2000 \\ 0.4x_1 + 0.15x_2 \geq 1500 \\ 0.2x_1 + 0.35x_2 \leq 1000, \\ x_1 \leq 9000 \\ x_2 \leq 6000 \\ x_i \geq 0 \end{aligned}$$

rsity/eng-3px3/Linear Optimization

$$\begin{aligned} \min_x \phi = c^T x & \quad \leftarrow \text{Objective function} \\ \text{s.t} & \quad \leftarrow \text{Constraints} \\ A_h x = b_h & \quad \leftarrow \text{Equality constraints} \\ A_g x \leq bg \leq 0 & \quad \leftarrow \text{Inequality constraints} \\ x_{lb} \leq x \leq x_{ub} & \quad \leftarrow \text{Variable Bounds} \end{aligned}$$

interest

Interest I is the compensation for loaning money.

interest rate

$i = \frac{I}{P}$. Thus $F = P(1+i)$

Simple interests

$I_{\text{each}} = P \times \frac{i}{\text{year}}$, total interest $I = I_{\text{each}} \times N_{\text{year}}$

$F_n = P(1+ni)$

Compound interests

$F_n = P(1+i)^n$

nominal interest rates

r is the equivalent yearly rate if interest is withdrawn so it doesn't compound. (i.e
of compounding periods per year)

effective annual interest rates

$i_{\text{eff}} = (1 + \frac{r}{m})^m - 1$

effective interest rates

how much interest do you accrue after a year if nominal rate is 12%? $F = P(1+i)^m = P(1+\frac{r}{m})^m$

continuous compounding

$F = Pe^{ry}$

net present value

$$\text{NPV} = \text{CF}_0 + \sum_{n=1}^{N} \frac{\text{CF}_n}{(1+i)^n}$$

where CF_0 is the initial cash flow, CF_n is the cash flow at the end of the n^{th} period, i is the *effective interest rate*

discount rate

Present value $PV = \frac{\text{CF}_t}{(1+r_d)^t}$, where CF_t is cash flow happening in t years in the future, and r_d is the discount rate.

sources: opportunity cost, inflation, risk, time preference, inflation, option premium

regular deposit: Future value $FV = A\sum_{k=0}^{n-1}(1+i)^k = A\frac{(1+i)^n - 1}{i}$ where A is the monthly, or time period, deposit.

fraction of last payment that was interest was $\frac{i}{1+i}$, principal of the last payment is $A = F_{\text{last}}(1+i)$

geometric series

$$\sum_{k=0}^{n-1} r^k = \frac{1-r^n}{1-r}$$

expected impact

the chance it happens multiplied by the impact it will have if it happens. $\text{E}[\text{NPV}] = \sum_i \text{NPV}(x_i)p(x_i)$

Then use this to create necessary mitigation

before-tax MARR: set MARR high enough to include taxes that need to be paid => for investment's gross profit
after-tax MARR: if tax is explicitly accounted for in the cash flows of the project, then MARR should be lower => for final investment decisions

$$MARR_{\text{after-tax}} = MARR_{\text{before-tax}} \times (1 - \text{corporate tax rate})$$

value calculations

Depreciation in year n $D(n)$ is the decline in book value over that year: $BV(n) = BV(n-1) - D(n)$

Salvage value SV is the book value at object's EOL: $SV = BV(N) = MV(0) - \sum_{n=1}^{N} D(n)$

Straight-line depreciation

spreads uniformly over useful life, SLD of a period $D_{\text{sl}}(n) = \frac{\text{Purchase price} - \text{Salvage value after N periods}}{\text{N periods of useful life}}$.

book value at end of n^{th} year: $BV_{\text{sl}}(n) = P - n \times \frac{P-S}{N}$

Declining-balance depreciation

different assets are classified into classes: $D_{\text{db}}(n) = BV_{\text{db}}(n-1) \times d(\text{depreciation rate})$, such that b
the end of a period $BV_{\text{db}}(n)$ is $BV_{\text{db}}(n) = P(1-d)^n$

given salvage value S and period of useful life N, depreciation rate $d = 1 - \sqrt[N]{\frac{S}{P}}$

Sum-of-years-digits depreciation

$D_{\text{syd}}(n) = \frac{N-n+1}{\sum_{i=1}^{N} i} \times (P - S)$

Unit of production depreciation

$D_{\text{uop}}(n) = \frac{\text{units produced of period}}{\text{life in \# of units}} \times (P - S)$

assumes a SLD but vs. # of units rather than time.

behavioural economics

invisible hand of the market: self-interest of individuals leads to the best outcome for society as a whole, in market economy, as rational actors are motivated by incentives.

perfect competition: wheat (control of price none, low barrier to entry, high # of producers, products are i
monopolistic competition: restaurants (control of price low, low barrier to entry, high # of producers, products are similar)
oligopoly: airlines (control of price high, high barrier to entry, few producers, products are similar)
monopoly: utilities (control of price high, high barrier to entry, one producer, unique product)

game theory, most notable The Prisoner's Dilemma

anti-trust legislation: prevent monopolies, promote competition, protect consumers

> behavioural economics: + psychology to look at reasons people make *irrational* decisions
>
> "bounded rationality": you don't have perfect information, and understand there's an opportunity cost to get it

law of demand and *ultimatum game*: people will pay less for a good if they can get it elsewhere for less, even if they value it more than the price they pay.

Cooperation: R. Axelrod's *The Evolution of Cooperation* propose a "strategy", what you do dependent on what the other person does.

PPF (production possibility frontier): trade-offs between two goods, given a fixed amount of resources.

risk aversion: people prefer a certain outcome to a risky one, even if the expected value of the risky one is higher. => assume that the given investment is loss, then calculate based on margin gains

supply and demand

market equilibrium: where supply and demand curves intersect, quantity demanded equals quantity supplied.
shift to right: greater demand, higher price, higher quantity. shift to left: lower demand, lower price, lower quantity.
factors of production: land, labour, capital, entrepreneurship
determinants of demand:

- price: quantity demanded Q_d falls when price P rises and vice versa
- prices of related goods: substitutes and complements

determinants of supply:

- price: quantity supplied Q_s rises when price P rises and vice versa
- factors of productions
- fiscal policies, taxes, regulation

elasticity: how responsive quantity demanded or supplied is to a change in price.

Surplus when $Q_s > Q_d$, shortage when $Q_s < Q_d$.

Elasticity of demand: $E_d = \frac{\%\Delta Q_d}{\%\Delta P} = \frac{|\frac{P}{Q_D}|}{|\frac{dP}{dQ_D}|}$

Elasticity of supply: $E_s = \frac{\%\Delta Q_s}{\%\Delta P} = \frac{|\frac{P}{Q_S}|}{|\frac{dP}{dQ_S}|}$

higher slope corresponds to lower elasticity: inelastic, lower slope corresponds to higher elasticity: elastic

Demand elasticity: $E_D < 1$ means if price increases by 5% then demand will decrease by less than 5%, inelastic.
$E_D > 1$ means if price increases by 5% then demand will decrease by more than 5%, elastic.

inflation

real vs. nominal

nominal value refers to actual cash flow at the time it hapens, real value refers to equivalent amount of value at reference time, converted using inflation rates.

real dollar $R = \frac{\text{CF}_n}{(1+r_i)^n}$, where CF_n is the nominal cash flow at time n, and r_i is the effective yearly inflation rate.

internal rate of return

the discount rate that results in a NPV of zero (break-even scenario)

$$\text{CF}_0 + \sum_{n=1}^{N} \frac{\text{CF}_n}{(1 + r_{\text{IRR}})^n} = 0$$

minimum acceptable rate of return

a rate of return set by stakeholders that must be earned for a project to be accepted

real vs. nominal MARR: real MARR is MARR if returns are calculated using real dollars, whereas nominal MARR is MARR if returns are calculated using nominal dollars.

$\text{MARR}_{\text{real}} = \frac{1+\text{MARR}}{1+f} - 1$ where f is the inflation rate

expected value for calculating stochastic to deterministic

of function $f(x)$ is $\text{E}[f] = \sum_i f(x_i)p(x_i)$ for discrete random variable x with probability distribution $p(x)$

of function $f(x)$ is $\text{E}[f] = \int_x f(x)p(x)dx$ for continuous random variable x with PDF $p(x)$

Normal distribution

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

`NORM.DIST(x, mean, stddev, cumulative)`: cumulative is `1` for CDF, `0` for PDF `NORM.INV(RAND(), 0.5, 0.05)`: draw values from a normal distribution with mean 0.5 and stddev 0.05

non-linear deterministic and stochastic models

mean value μ_x of a random variable x is its own expected value $\text{E}[x]$, variance σ_x^2 is the expected value of the squared deviation from the mean $\text{E}[(x-\mu_x)^2]$, and stddev σ_x

central limit theorem

sample size becomes large enough, the distribution of the sample mean will be approximately normally distributed, regardless of the distribution of the population, using Monte-Carlo simulation

> Expected value of linear and nonlinear functions: suppose x and y are independent random variables with means μ_x and μ_y, and variances σ_x^2 and σ_y^2, then $E[x^2] = \sigma_x^2 - \mu_x^2$, $E[xy] = \int\int xyp_xp_ydxdy = \int xp_xdx \int yp_ydy = \mu_x\mu_y$

Dealing with 12 months per year: saying outcomes over a year should be **normally distributed** (CLT), with a mean given by expected value of monthly outcome and stddev given stddev of outcome divided by square root of the # of rolls ($\sqrt{12}$)