2FA3 - Assignment 1

Submitted via Avenue. Due Feb 16th, 11:59pm. Note: I indicate what each question if worth by square brackets, i.e. [k].

1. For each statement below, state if it is true of false, and explain why. The explanation does not need to be a formal proof, but the argument should be sound.[6]

   (a) If L_1 is regular and $|L_1| = k$ and L_2 is non-regular, then $L_1 \cap L_2$ is regular.

   (b) If L_1 is regular and L_2 is non-regular, then $L_1 \cup L_2$ is regular.

   (c) $\forall L_1$ such that L_1 is a non-regular language, $\exists L_2$ such that L_2 is regular and $L_1 \subseteq L_2$.

2. Create a DFA M, such that:

   (a) M accepts all strings which begin with b but do not contain the substring bab. [2]

   (b) $\mathcal{L}(M) = \{a^i b^j c^k \mid i + j + k \text{ is a multiple of 3 }\}$, $\Sigma = \{a, b, c\}$ [3]

   (c) $\mathcal{L}(M) = \{x \mid \text{There are at least two a's in the last three characters of } x\}$

3. Via product construction, create a DFA M, such that

   $$\mathcal{L}(M) = \{a^n b^m \mid n \text{ or } m \text{ is a multiple of } 3\}$$

   First create two machine: one where n is a multiple and one where m is a multiple of three. Then create the "union" machine. When I say create two machines, I mean an M_1 and M_2 such

   $$\mathcal{L}(M_1) = \{a^n b^m \mid n \text{ is a multiple of } 3\}$$
   $$\mathcal{L}(M_2) = \{a^n b^m \mid m \text{ is a multiple of } 3\}$$

   [5]

4. Create an NFA which accepts all strings in which the third last character is an a. Then via subset construction, create an equivalent DFA. Show all your work. [5]

5. Create a sound argument that the concatenation of two regular languages is also regular. Specifically, if L_1 and L_2 are regular then $L_1 L_2$ is also regular. This does not need to be a formal proof, but the argument should be very convincing. Hint: if L_1 and L_2 are regular you can make "machines" for them; how would you make a "machine" for $L_1 L_2$.